CLASSICS

Introduction to Classics
Context, Relevance, and Impact of the 1970 *Nature* Article of C.V. Vishveshwara

For the first half of the 20th century, the international scientific community, including Einstein, was frequently confused about the initial predictions of Einstein's new theory of space, time, and gravitation known as General Relativity (1916). The first exact solution to this theory was the spherically symmetric static Schwarzschild solution, which appeared to have a puzzling apparent singularity at its center and might have been unstable, leading to strong doubts about its significance for physics or relevance in nature. Gravitational radiation theory was also particularly confused, with some people claiming that radiating systems could gain energy by radiation-reaction, leading to run-away solutions, while others doubted that radiation could carry any energy, momentum, or information or have any meaningful physical reality. C.V. Vishveshwara contributed to important progress on many of these mysteries and confusions by his creativity and imagination.

When I started my senior year at Columbia College in the fall of 1961, I enrolled in a graduate course in Methods of Mathematical Physics taught by Robert Fuller. Together with a few undergraduate physics majors, I sat in the front of the lecture room, and we frequently dominated the class discussions. In the rear of the room, a few beginning graduate students usually gathered together, intimidated by our apparent facility with the subject. Among these reticent scholars was C. V. Vishveshwara ("Vishu"). I have no recollection that we undergraduates ever had a real interaction with the more senior graduate students. As it turned out, after serious discussions with Fuller, Vishu switched from his initial interest in particle physics to gravitation and moved on to join Charles Misner at the University of Maryland as a thesis advisor.

I went on to Stanford for the next two years, where I had a course in General Relativity with Prof. Manahem Schiffer in the Mathematics Department, and I was mesmerized by the role of geometry in this branch of theoretical physics. I took a reading course in cosmology with Leonard Schiff in the Physics Department. Schiff, was one of only two people who had suggested new tests of General Relativity since Einstein (Irwin Shapiro at MIT was the other). One day, he volunteered what at that time was absolutely correct advice for prospective students: "*Stay away from Relativity! It is the province of mathematicians. Experimentation is impossible. It is no place for a physicist to venture*." (A few years ago, I heard from Misner that Kip Thorne and Igor Novikov had also gotten similar advice around this time from their advisors.) However, I was unreasonably fascinated with the subject and asked around for suggestions of someone who would be the best mentor for a thesis in this field, and kept hearing the same answer—Charlie Misner at the University of Maryland. So, in the fall of 1964, I left Stanford for Maryland, joined Misner's close group of doctoral students (seven, including Vishu and me), and started my thesis work on gravitational radiation. This time around, I quickly met and formed a deep and long-lasting friendship with Vishu.

For his Doctoral thesis with Misner at Maryland in the 1960s, Vishu carried out important research on the stability and geometrical characterization of the simplest solution (i.e., the Schwarzschild solution) to the 10 coupled nonlinear partial differential equations that Einstein provided. This solution characterized the concentration of gravitational energy that was to be named a "black hole" by John Wheeler in 1967. Vishu completed his PhD work in 1968, and moved on soon after. For his postdoctoral efforts, he initiated the early use of computer simulations to probe the dynamical behavior of a black hole when excited by radiation. He published his pathbreaking new results in an important article (C. V. Vishveshwara, Scattering of gravitational radiation by a Schwarzschild black hole, *Nature*, 227, pp.936–937, 1970). This work is now recognized as a classic contribution to theoretical gravitational physics and was finally verified experimentally by the discovery of gravitational wave emission from the coalescence of binary black holes in 2015 (GW150914), leading to the award of the Nobel Prize in Physics to the leaders of the thousand scientist LIGO collaboration, B. Barish, K. Thorne, and R. Weiss, two years later.

How was Vishu rewarded by the scientific community for his proof that black holes were not only stable, but also for his unexpected result of their tangible reality by the insight that black holes ring like a bell when struck? He has left an illuminating personal account of this experience:

> "*It was the best of times, it was the worst of times. For me, that is. This was in the year 1969. I was working at the Institute for Space Studies in New York on a National Research Council Fellowship. I was studying the scattering of gravitational waves by black holes through computer simulation in order to find out whether*

the black hole left its imprint in some way on the scattered wave. This was done by bombarding the black hole with Gaussian wave packets. When the width of the Gaussian was comparable to or less than the radius of the black hole, there emerged a decaying wave pattern, later to be called the quasinormal mode of the black hole. As is well known, a lot of work has been done on the black hole quasinormal modes since then. Today they are considered to be a means of detecting both black holes and gravitational waves. My original work on the quasinormal modes was terribly exciting to me. And that is the best-of-times part of the story."

"*Now for the worst-of-times aspect. Unfortunately, no one at the Institute seemed to be interested in this kind of research. After all, neither black holes nor gravitational radiation had been detected, but only theoretically predicted. Why should anyone spend time investigating the interaction between two unobserved entities perhaps of doubtful existence? The consequence of this attitude, which was not at all uncommon in those days, was rather disconcerting to say the least. My contract was renewed for only three months instead of the normal one year period.*"

– The Engelbert Experience: Pathways From the Past, C. V. Vishveshwara

Unfortunately, Vishu was temporarily caught up in a national job crunch for physicists. The job market in the United States was saturated due to the overproduction of physicists. This was caused by previous government overstimulation, leading to chaos and imbalance between the number of applicants and the physics jobs available at universities. His experience was shared by an emerging generation of young physicists, who found themselves applying to several hundred different job advertisements with little response.

Vishu managed to overcome this temporary setback and subsequently went on to work at New York University and the University of Pittsburgh before returning to India, where he continued his research at the Raman Research Institute and later at the Indian Institute of Astrophysics. He finally turned his efforts and creative energies to education and outreach as the Founder Director of the Jawaharlal Nehru Planetarium, Bengaluru.

S. Chandrasekhar ("Chandra") visited Vishu when he was at NYU, and was very much interested in the stimulating results uncovered by Vishu's pioneering computer simulations. Chandra later went on compute the first few characteristic frequencies excited in a Schwarzschild black hole (S. Chandrasekhar and S. Detweiler, The quasi-normal modes of the Schwarzschild black hole, *Proc. R. Soc. London, Ser. A*, 344, pp.441–452, 1975). The authors analyzed the differential equations for perturbations of a black hole, corresponding to incoming waves at the horizon and outgoing waves at infinity. Following them, extensive literature extending these calculations to many more quasinormal modes has developed. This subject is comprehensively reviewed in K. D. Kokkotas and B. G. Schmidt, Quasi-normal modes of stars and black holes, *Living Rev. Relativity*, 2, 2, 1999. Following the important insights from Vishveshwara's computer experiments, theorists have carried out an extended program of analytic calculations and numerical simulations of perturbations of black holes of more general character. What has emerged is the understanding that the gravitational radiation emitted from black hole oscillations is dominated by certain characteristic frequencies that are independent of the processes creating these oscillations. Rather, they are completely determined by the parameters characterizing the final black hole (mass, charge, and angular momentum). Moreover, for black holes with mass tens of times that of our sun, these frequencies are within the capabilities of existing Earth-based gravitational radiation detectors and so are susceptible to experimental detection and verification.

Today, after many decades of theoretical effort, state-of-the-art computer simulations have improved to allow full 3-D modeling of the dynamic behavior of merging pairs of black holes and the consequent gravitational radiation that will arrive on Earth. Most importantly, we have actual experimental data available corresponding to such strong-field processes. Thus, we are now finally able to extract and compare both the detailed predictions of Einstein's nonlinear theory and the corresponding experimental observation. Efforts in this field are, therefore, currently directed to improving the precision of both predictions and observations, and to verify their validity with ever greater precision. A current example of the very sophisticated analysis used to extract the power, spin, and mass of the final black hole from the dominant quasinormal modes of the gravitational radiation field (seen in the lab or in computer simulations) can be found in L. M. Zertuche *et al.*, *Phys. Rev. D*, 105, 104015, 11 May 2022.

A diverting offshoot of Vishu's insights is a recent paper dedicated to him, published by his daughter Smitha Visheshwara, a theoretical condensed matter physicist, and her collaborators. In this work, the authors propose that the quantum Hall system can be a platform for exploring black hole phenomena. They show that both systems can be elegantly modeled by a simple quantum system, the inverted harmonic oscillator, which also exhibits quasinormal mode oscillations (S. S. Hegde *et al.*, *Phys. Rev. Lett.*, 123, 156802, 11 October 2019.).

It is wonderful that in 2015, Vishu was able to witness the LIGO project's discovery of gravitational radiation, produced by a pair of coalescing black holes, with the signature quasinormal mode oscillations in its final waveform, 45 years after his original prediction. This is something that he had dreamed of and enthusiastically wished for all his adult life, and in the end, he had the great pleasure of enjoying. Now, LIGO observations at greater sensitivity, and improvements in data analysis algorithms by theorists, will enable the use of quasinormal mode observations in the near future for measurements of the spin of the underlying black hole with high precision. This is something that has so far proved a challenge through any other direct observation technique.

Finally, it is impossible to resist mentioning Vishu's charming use of cartoons and caricatures in his lectures, publications, and outreach efforts. So, I will close with a few examples in which he collaborated with the artist B. Gujjarappa, who did the final artwork based on Vishu's preliminary sketches and conceptual design. These three portraits capture impressions of figures from his student days at the University of Maryland as they matured. They are the late Charles Misner (June 13, 1932—July 24, 2023), me, and Vishu, as he saw himself.

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Scattering of Gravitational Radiation by a Schwarzschild Black-hole

THE discovery of pulsars and the general conviction that they are neutron stars resulting from gravitational collapse have strengthened the belief in the possible presence of Schwarzschild black-holes—or Schwarzschild horizons—in nature, the latter being the ultimate stage in the progressive spherical collapse of a massive star. The stability of these objects, which has been discussed in a recent report[1], ensures their continued existence after formation. Inasmuch as the infinite redshift associated with it and its behaviour as a one-way membrane make the

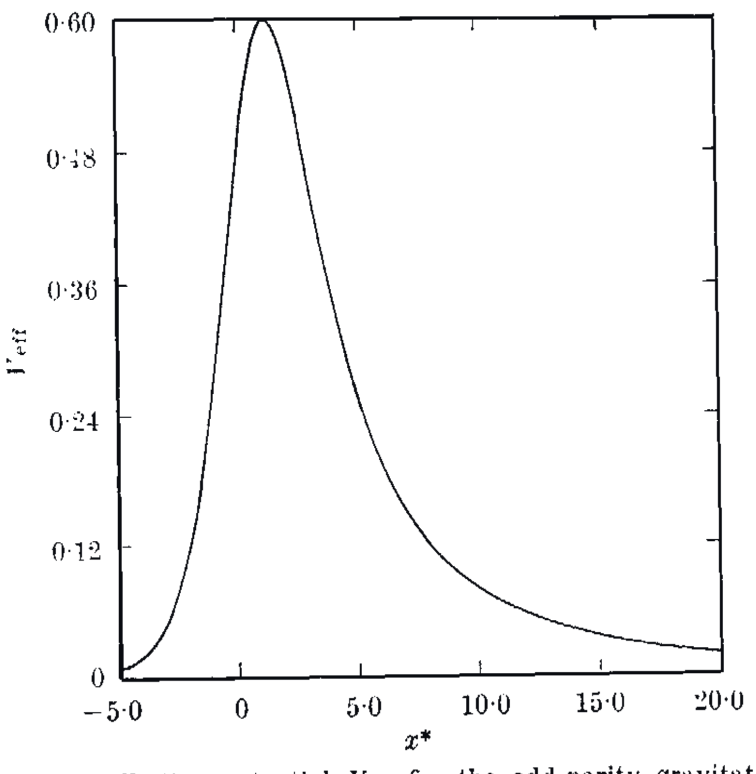

Fig. 1. The effective potential V_{eff} for the odd-parity gravitational waves of the lowest mode $l=2$ plotted against x^*.

Schwarzschild horizon at once elusive and intriguing, it is important to explore theoretically all possible modes in which the presence of such a black-hole manifests itself. In what follows, we present a partial summary of some results obtained from an investigation of the scattering of gravitational waves by a Schwarzschild horizon.

To begin with, a spherically symmetric mass distribution is assumed to have collapsed into its Schwarzschild surface in the infinitely remote past. The scattering of gravitational waves by this configuration can be examined employing the perturbation techniques developed by Regge and Wheeler[2]. Retaining the notation of the authors and concentrating on the odd-parity waves, the perturbed Schwarzschild exterior line element in the Regge–Wheeler canonical gauge can be written as

$$ds^2 = -(1-2m/r)dt^2 + (1-2m/r)^{-1}dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2) + (h_0(r)\,dt\,d\varphi + h_1(r)\,dr\,d\varphi)\exp(-i\omega t)\sin\theta \frac{dP}{d\theta} l(\cos\theta)$$

where ω is the frequency of the gravitational waves. The Einstein empty-space field equations computed to first order in the perturbations[3] yield the following differential equations for the radial functions h_0 and h_1

$$\frac{d^2Q}{dx^{*2}} + (k^2 - V_{eff})Q = 0 \text{ with } V_{eff} = (1-1/x)(l(l+1)/x^2 - 3/x^3)$$

and

$$h_1 = \frac{i}{k}\frac{d}{dx^*}(xQ)$$

where we have defined

$$x = r/2m,\; x^* = x + \ln(x-1),\; k = 2m\omega, \text{ and } Q = (1-1/x)h_1/x$$

The exterior from $r = 2m$ to ∞ corresponds to the range of x^* from $-\infty$ to $+\infty$. The motion of the gravitational waves in this space is governed by the effective potential V_{eff} produced by the collapsed mass. The effective potential for the lowest possible mode $l = 2$ is plotted against x^* in Fig. 1, and the general behaviour of the potential for any higher value of l is the same, it is positive and goes to zero asymptotically as x^* approaches $\pm\infty$ and attains a maximum in between. From the Schrödinger form of the wave equation for Q and from the shape of the potential, it is evident that the scattering problem here is formally the same as that encountered in quantum mechanics for a one-dimensional potential barrier. A wave coming from spatial infinity is partially reflected by the effective potential, so that at large values of x we have both incoming and outgoing waves. On the other hand, as the Schwarzschild horizon acts as a sink for the radiation, there will be only waves entering the $r = 2m$ surface. Consequently the suitable asymptotic boundary conditions are $Q_\infty = A(k)e^{-ikx^*} + B(k)e^{ikx^*}$ and $Q_\infty = C(k)e^{-ikx^*}$ for x^* approaching $\pm\infty$ respectively. In analogy with the quantum mechanical problem we can define the reflexion and transmission coefficients $R = |B/A|^2$ and $T = |C/A|^2$. A fraction R of the incident wave escapes to spatial infinity and is accessible to a distant observer, whereas a fraction T of the radiation is absorbed by the black-hole and thereby lost in the process. An analytical integration of the equation for Q leading to the computation of R and T has been impossible in practice and recourse had to be taken to numerical integration. This has been carried out—as have further computations to be discussed later—for the lowest mode $l = 2$ using a computer. Fig. 2 shows the plot of R against k^2. In the limit of zero frequency the reflexion coefficient approaches the limit 1 independent of the scattering mass, and so in this limit no information about the latter is forthcoming. Nevertheless, the rate at which the reflexion amplitude B/A, and so R, decreases as a function of the frequency should be perceptible when a sufficient range of frequencies is included and, because this rate clearly depends on the scattering mass, the presence of the latter is "coded" into the outgoing radiation. This leads us at once to the

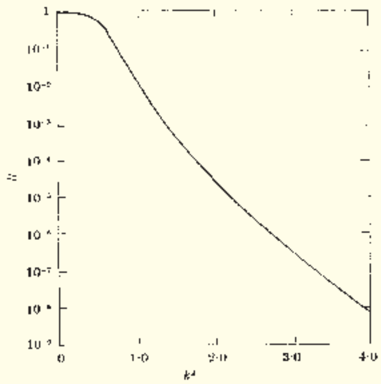

Fig. 2. The reflexion coefficient, R as a function of k^2, the square of the frequency, for odd-parity mode $l = 2$.

physically more interesting phenomenon of the scattering of wave packets. By linear superposition we obtain an incoming wave packet at infinity with the spatial profile

$$\psi_{in}(x) = \int_{-\infty}^{+\infty} f(k)e^{-ikx}\, dk$$

and the corresponding outgoing or reflected packet

$$\psi_{out}(x) = \int_{-\infty}^{+\infty} (B/A)\, f(k)e^{ikx}\, dk$$

The latter travels out without spreading and is received by the distant observer. We choose the Gaussian function

$$f(k) = \frac{1}{2\sqrt{\pi\alpha}}\, e^{-k^2/4\alpha}$$

in order to obtain a simple model for an incoming wave packet, that is, $\psi_{in} = e^{-\alpha x^2}$. As the parameter α, which measures the width of the wave packet, is varied, some interesting features emerge. For low values of α ($\alpha \lesssim 0{\cdot}01$), that is, for very broad incoming packets or equivalently for $f(k)$ sharply peaked at zero frequency, the reflected packet is practically unaffected. But, as the parameter α is gradually increased, ψ_{out} develops distinct maxima and minima that increase in number progressively, while their relative spacing undergoes a continuous change. As the parameter α, however, approaches approximately the value 1, that is, for a width of about the Schwarzschild radius, the process reaches a limit. Beyond this value of α, as the packet is made thinner, the outgoing packet will cease to develop new peaks and the relative spacing of these peaks will remain unaltered. In other words, any higher frequencies added to the original packet will have negligible effect on the scattered packet owing to their almost total absorption by the black-hole. As long as the incoming packet is spatially sharp enough, the reflected packet will manifestly carry information about the scattering mass. Fig. 3 shows an example of the "saturated" pattern corresponding to $\alpha = 1$. The spacing between consecutive peaks and, consequently, the lag in their arrival times are measures of the scattering mass, as the spacing in the actual radial distance is given by $\Delta r = 2m\Delta x$.

The total energy carried by a wave packet $\psi(x)$ at spatial infinity can be computed by adapting a method used by Edelstein[4]. The result of this computation is that, for any mode l, the energy of the wave packet is given by

$$E = (c^4/32\pi G)\,(l-1)l(l-1)\,(l+2)\int(\psi(x))^2\, dx$$

where the integration is carried over the spatial extent of the wave packet. So the fraction of incident radiation scattered by the black-hole and reaching spatial infinity

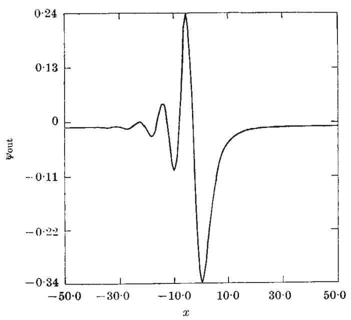

Fig. 3. The outgoing wave packet $\psi_{out}(x)$ at spatial infinity corresponding to the incident Gaussian wave packet $\psi_{in}(x) = e^{-ax^2}$ with $a = 1$.

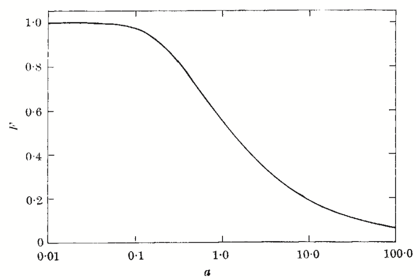

Fig. 4. The fraction F of the incident energy carried by the scattered outgoing wave packet at spatial infinity plotted as a function of the parameter a.

is the ratio of the energies carried by the incoming and the outgoing wave packets and is readily computed as

$$F = \frac{\int(\psi_{out})^2\, dx}{\int(\psi_{in})^2\, dx} = \left(\frac{2a}{\pi}\right)^{1/2} \int(\psi_{out})^2\, dx$$

In Fig. 4 the fraction F is plotted as a function of the width-parameter a. For an incident wave packet, the width of which is about a Schwarzschild radius ($a \simeq 1$) approximately half the total energy is scattered and the rest absorbed by the black-hole.

We have confined ourselves so far to some results concerning the scattering of odd-parity gravitational waves of angular momentum $l = 2$ by a Schwarzschild black-hole. The mathematical and numerical details omitted here, as well as the scattering of higher l modes, even-parity waves, scalar gravitational waves and finally electromagnetic waves, will be discussed elsewhere in a separate and more detailed paper.

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Some of the caricatures made by B. Gujjarappa, in collaboration with C. V. Vishveshwara.

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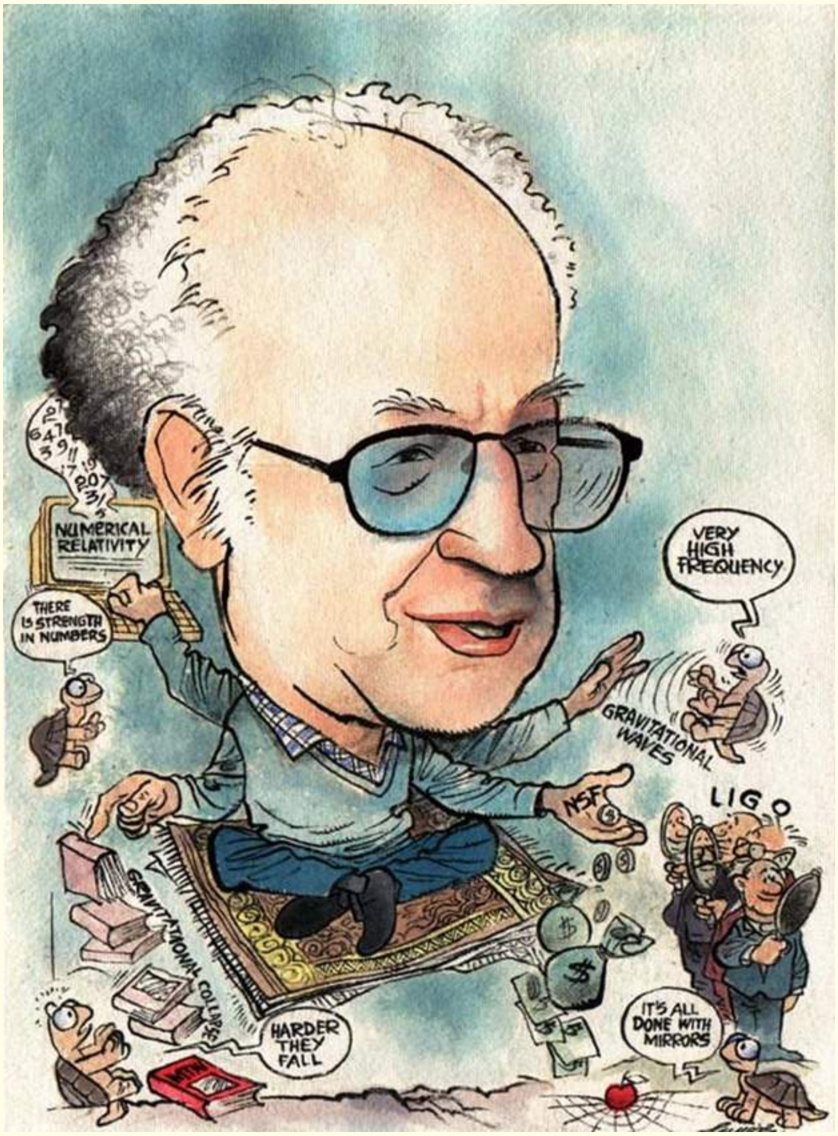
C. W. Misner

The Master Mixer


NUMERICAL RELATIVITY
THERE IS STRENGTH IN NUMBERS
VERY HIGH FREQUENCY
GRAVITATIONAL WAVES
LIGO
NSF
GRAVITATIONAL COLLAPSE
HARDER THEY FALL
IT'S ALL DONE WITH MIRRORS
MTW


HIGHER DIMENSION
GHOST VILLE 15
UNDER COVER ORGANIZATION
• SCHWARZSCHILD STABLE
• KERR WHIRLPOOL
COLLECTED
• BASE CAMP
• BLACK HOLE HUNT